DECODING THE SPITFIRE

In the beginning there was the ellipse

"I don't care a damn what shape it is as long as we can get the guns in!"

The above statement, or one similar to it, as mentioned in various books, may have been made by R. J. Mitchell relating to the well-known elliptical shape of the Spitfire wing. According to Lance Cole, author of "Secrets of The Spitfire", Shenstone had been quoting Mitchell, *"I don't give a bugger whether it is elliptical or not, so long as it covers the guns."*

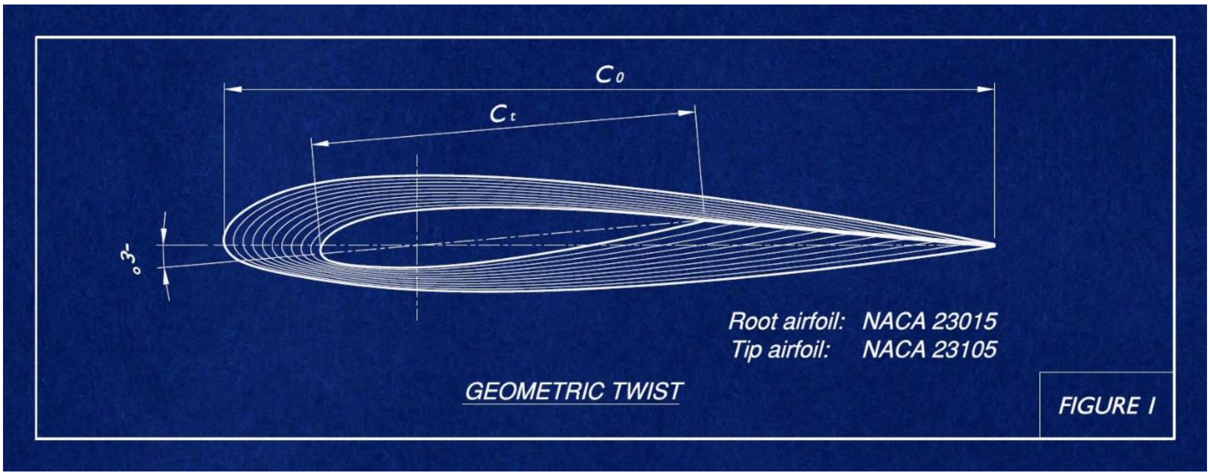
R. J. Mitchell led his design team with an autocratic hand that ruled on every major decision. Why then would R. J. Mitchell make a comment like that? Is it because he had no opinion on elliptic wings and didn't care whether it was elliptical or trapezoidal? Not in this writer's opinion. The reason he might have said that is not because the wings were to change from trapezoidal to elliptical, but rather because the wings were to change from elliptical to not so elliptical.

Time was running out and the last minute decision to populate the wings with 8 instead of 4 guns necessitated short cuts to be made. It was 1934 and there were no computers to quickly modify the designs and drawings. What was the original ellipse like and what were the thought processes of the aerodynamicist, the draughtsman while shaping the wing? What was the short cut and how was it done? Would it be possible to reproduce the steps and errors and come up with an identical plan-form just by studying the drawings? This attempt will be made here.

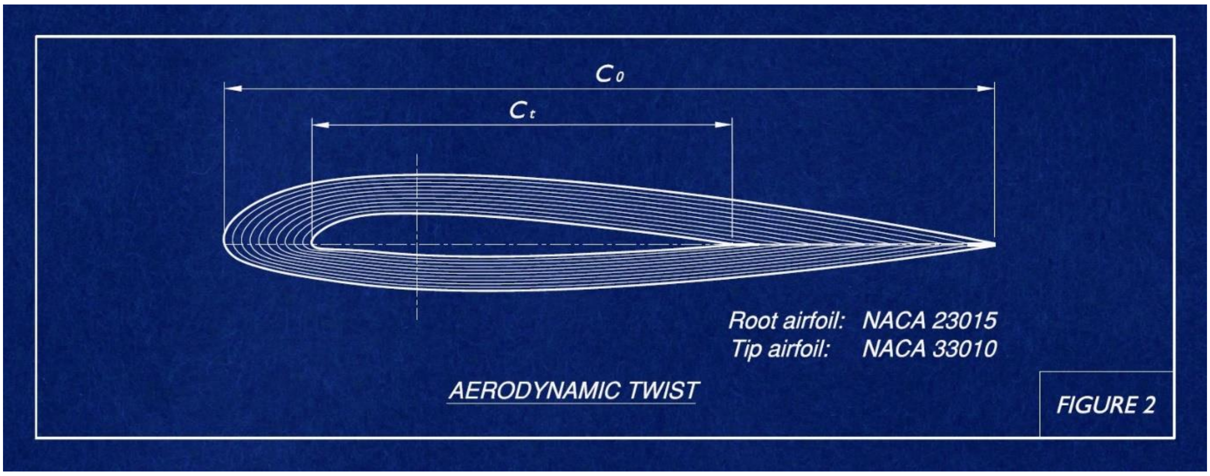
The shape of the spitfire wing is a complete mystery and its derivation is nowhere mentioned and no one is taking credit for the short cut to its final from. Did the drawing office mess up? The draughtsman of the day kept his checks and balances quite accurately as seen on Supermarine drawings. Was there a deliberate retrofitting an already elegantly laid-out elliptical planform?

Firstly, let us define the word elliptic and how it relates to the Spitfire wing shape. The elliptic shape of the Spitfire wing will be treated purely as a geometric characteristic here. A wing with an elliptic planform, provided it has identical airfoils along its span with no twist, will produce an elliptic loading. (This is not exactly true, however, as the Reynold's number is not constant due to different airfoil chords, but this is not the issue here and we will assume a constant Reynold's number.) The Spitfire had evolutive airfoils of the NACA 2200 family, but they are not strictly identical. Also, the wing was twisted for better stalling characteristics which further offset an ideal elliptic loading.

It is important to distinguish between an elliptic shape and an elliptic loading. A straight tapered



wing, for example, given the correct geometric twist (Figure 1) and/or airfoil sections along its span

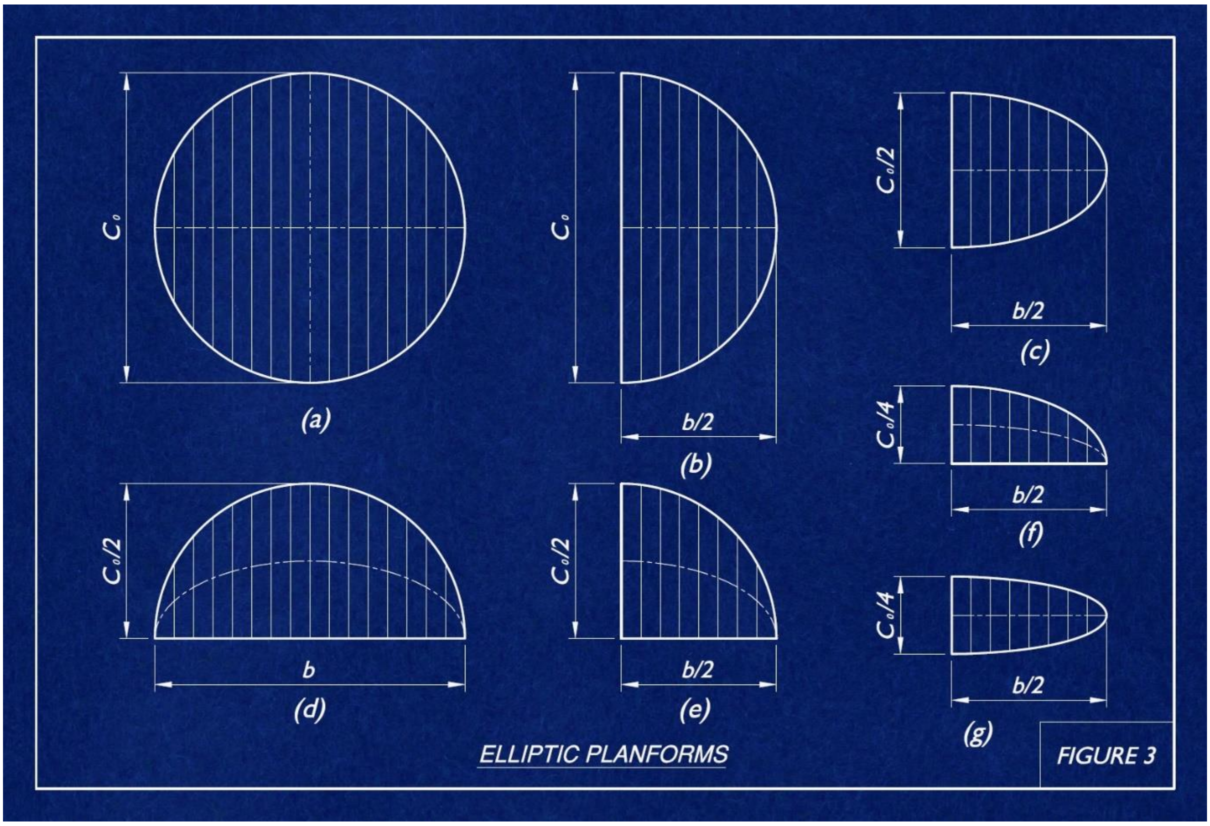


(aerodynamic twist, see Figure 2) can give elliptic loading. (Although at a single speed and altitude).

The elliptic planform was chosen for space requirements and the aerodynamic characteristic was of secondary importance, the elliptic loading being a bonus for minimal induced drag and other factors. According to Cole, Shenstone tweaked the wing geometry for maximum aerodynamic efficiency. When comparing the Spitfire wing planform with a perfect ellipse, however, the two are nearly identical except for a maximum difference of 2 inches near the tip. This will hardly produce any measurable change on the aerodynamic performance all other factors being equal. The aileron gaps are enough to more than offset this difference, for example.

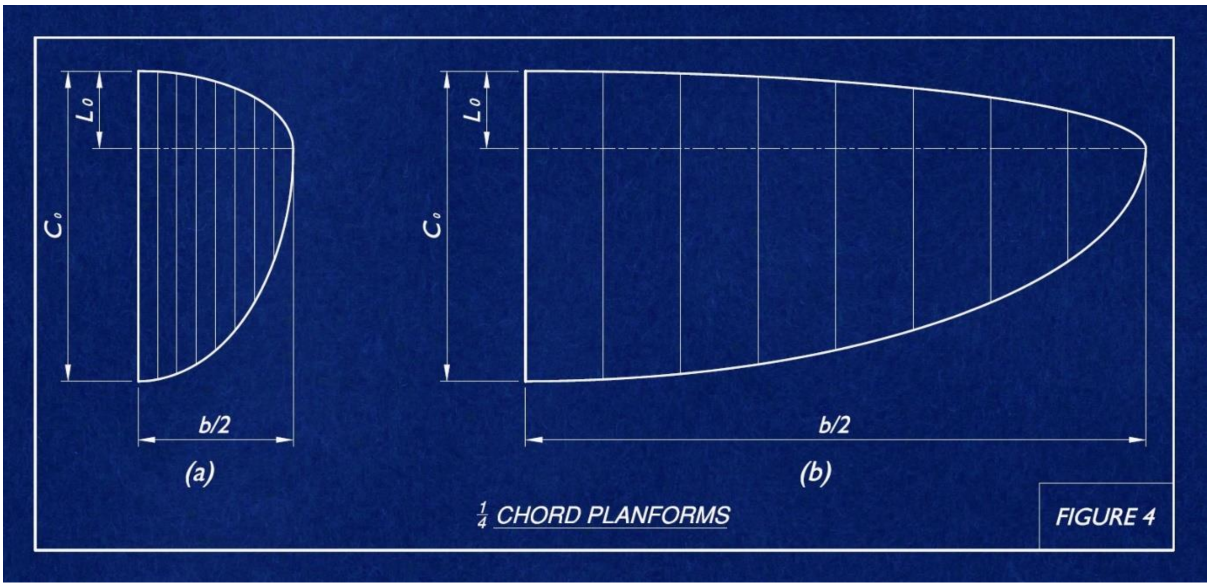
The ellipse is an elegant and an extremely simple curve to lay out. Moreover, if it is a wing planform it would be easy to calculate the cord length at any span. To deviate from this shape slightly, by approximating with an arbitrary, hand-drawn curve would be ludicrous by any standard. The Spitfire wing area is 242 sq. feet compared to truly elliptical shape of 242.7 sq. feet. The 100 square inches of area hardly justifies the trivial deviation from an ideal ellipse. This is the area of an A4 sheet of paper. Many of the wing production drawings had been accurately drawn to a large scale and the headaches and inconvenience of joining the near ellipse curves would have been considerable in the drawing office. One would not be able to use arbitrary spans to lay out the chord lengths but had to use a specific set of points along the span, then interpolate graphically for the exact location, to give an example.

An ellipse is a ‘squashed’ circle, and in fact the circle is a special form of the ellipse, just like the square is a special form of the rectangle. If all the vertical chords of a circle were scaled by a constant, and the horizontal distances kept the same, they would be circumscribed by an ellipse.

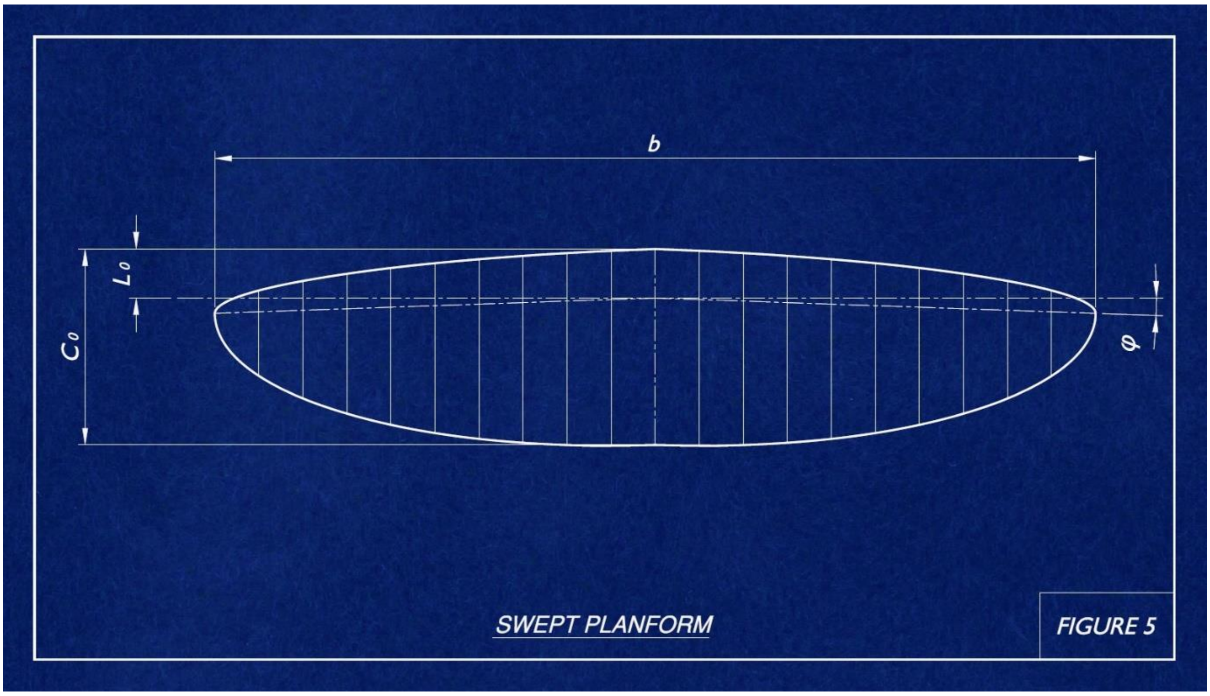


In Figure 3 planforms (a) to (g) all have an elliptical chord distribution. The chain line in (d) cutting the chords in half is elliptical and so is the one in (e). If the chain line in (e) were straightened it would form the elliptic form shown in (c). If the ellipse in (c) were halved horizontally it would result in (f). If the chords in (f) were bisected by the chained line and the line straightened it would result in an ellipse shown in (g).

Instead of bisecting the chords, the cutting line may divide the chords in other ratios. For example, if chords of the semi-circle in Figure 3 (a) were to be placed at their quarter length about the horizontal bisector it would result in the form shown in Figure 4 (a) and it would also have the same area. Notice that the semi-circle is now made of a top and bottom quarter ellipses.



Consider an ellipse of similar eccentricity to the Spitfire wing with the ratio of b/2 to C at 4:1. The ratio of b/2 to C in Figure 4 (a) is 1:2. Multiplying the horizontal distances between the vertical chords by 8 will result in Figure 4 (b). This resulting shape also consists of two quarter ellipses and the chord distribution will again be elliptical.



The line about which the chords are positioned may now be called the planform axis, or wing axis. It can be straight, swept forwards or backwards, or curved. A backward swept planform is shown in Figure 5.

In case of sweep or a curved axis the upper and lower bounding curves are no longer elliptical. If the axis is a straight line, or if it is a curve which can be expressed with an equation, the bounding curves may themselves be expressed as equations. The resulting planform has an elliptic chord distribution, although the leading and trailing edges are not ellipses. The planform in Figure 5 (or a planform with a curved or different sweep axis) has the same area as the planform in Figure 4 (b). This is analogue to the area of a parallelogram.

For a planform with elliptical chord distribution the local chord is

$$C = C_0\sqrt{1-4\left(\frac{y}{b}\right)^2} \qquad \textit{Equation 1}$$

The area is

$$A = \frac{\pi}{4} b C_0 \qquad \textit{Equation 2}$$

The Spitfire has a wing span of 445 inches with zero dihedral. The root chord is 100 inches. If its wing planform was truly elliptical the total wing area would be

$$A = 0.7854 \cdot 445 \cdot 100 = 34,950.3 in^2 = 242.7\ ft^2$$

Any chord along the semi-span may be easily found. For example, the local chord at 178 inches from centre line of aircraft is

$$C = 100\sqrt{1 - \left(2\frac{178}{445}\right)^2} = 60 in$$

On the Spitfire wing this chord is 59.77 inches.

Why this tantalising difference of .23 inches? For what possible reason would anyone deviate .23 inches in 15 feet? For the metric mind this is 6 mm in 4.5 meters.

A possible scenario

A summary of chronological events leading to the Spitfire prototype is given below.

October 15 1933	Rolls Royse first P.V.12 engine run.
February 19 1934	First flight of type 224 (Specification F.7/30) - 600-h.p. Rolls-Royce engine, piloted by "Mutt" Summers.
September 1934	Drawing No. 300 00 sheet 2 - A tapered wing with rounded tips and swept-back main spar. The Air Ministry enters negotiations between with the Colt Co. for a licence to produce the .303 machine guns.
First week October 1934	First Ellipse for revising type 224. Drawing No. 30000 sheet 11 - An early elliptical wing proposal for evaporatively cooled Rolls-Royce Goshawk engine. The ellipse was curved backwards with a back-swept spar.
November 6 1934	Vickers (Aviation) Ltd decided on to finance type 300 (a redesign of type 224 to Specification F.5/34 which the Air Ministry rejects) powered by the new P.V. 12 engine.
November 16 1934	The RAF publishes the requirement for eight guns.
Mid November 1934	Decision made to go elliptic.
Beginning December 1934	Drawing NO. 300 00 Sheet 12 - Final Elliptic Shape of type 300.
December 1934	Construction of Prototype K5054 began.
January 3 1935	The Air Ministry accepts the new design (with 8 guns and a P.V.12 Merlin engine) and writes specification F.37/34 around the new aircraft.
March 1935	Final wooden mock-up conference.
April 2935	Specification for Day and Night Fighter with 8 guns, F.10/35 received by Supermarine.

March 5 1936 — Maiden flight at Eastleigh Aerodrome powered by a Merlin “C” engine, providing 990 hp.

Supermarine type 224 had a wingspan of 550 inches or 45’ 10”. The chord was 100 in at the root and 65 in at the tip. With rounded off tips this gave a wing area of 295 sq. ft. It was decided to revise this design.

A number of revisions have been made. Probably the first was with the root chord was reduced to 90 inches with a taper ratio of 2.5:1 and the span reduced by 126 inches to 35’ 4”. With rounded tips this gave a wing area of 184 sq. ft. This was drawing No. 300 00 SHT 2.

Retracing the original ellipse

One has to start somewhere. This will be an assumption that there had to have been an original ellipse for the planform.

There had to have been a prototype ellipse for lofting the airfoil sections. One clue can be found in type 300 wing drawings and it is the wing-tip radius of 9 inches. This can be a vital clue to the possible ellipse eccentricity ratio, as will be seen later. For a root chord of 90 inches and a span of 450 inches, the resulting ellipse will have a tip radius of 9 inches – exactly. This can be shown to be

$$R_{tip} = \frac{{C_{root}}^2}{2b} \qquad \textit{Equation } 3$$

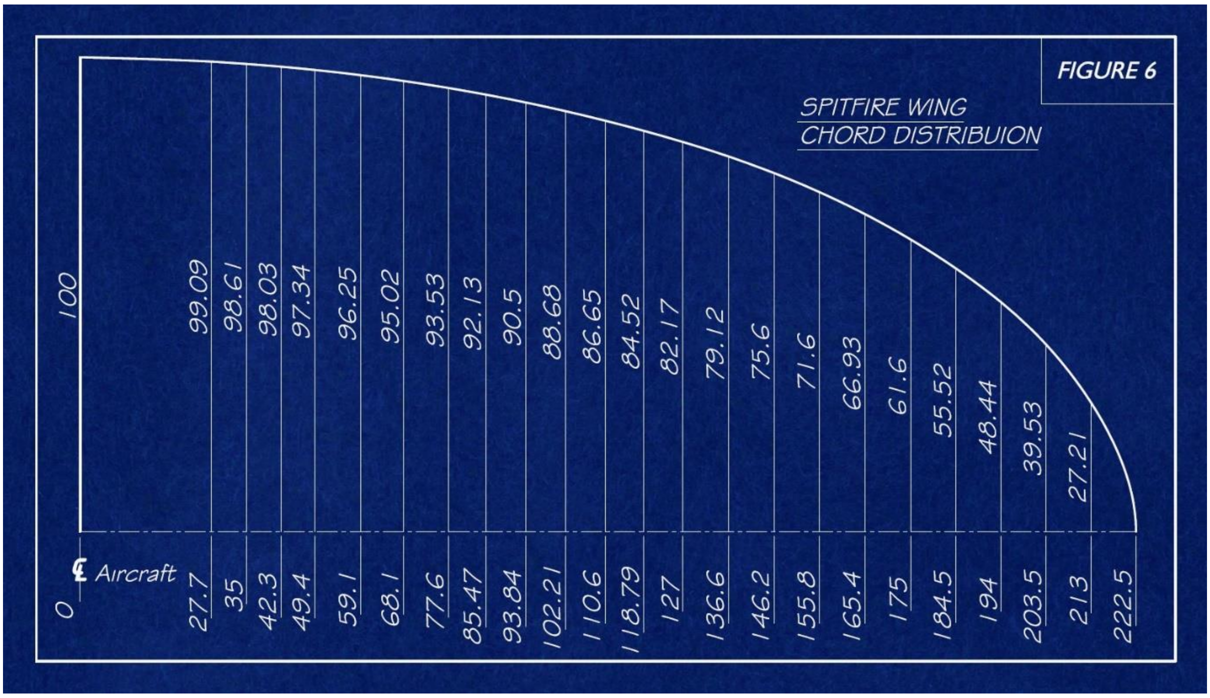
for an elliptic shape planform. This might well have been the starting ellipse.

Let us now consider the chord distribution as per Supermarine Drawing No. 337 08 SHT 8 and given in Table 1, and illustrated in Figure 6 where a true ellipse is shown dotted.

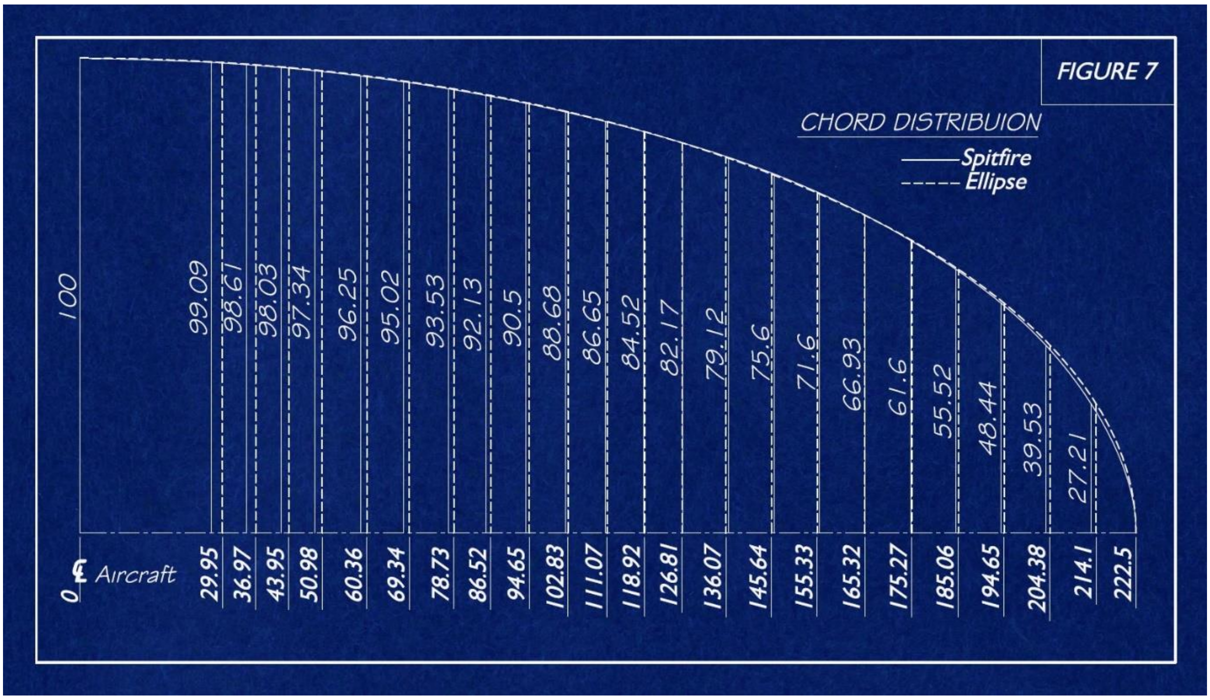
TABLE 1
Chord Distribution

STA	y in.	C in.
0	0	100
2	27.7	99.09
3	35	98.61
4	42.3	98.03
5	49.4	97.34
6	59.1	96.25
7	68.1	95.02
8	77.6	93.53
9	85.47	92.13
10	93.84	90.5
11	102.21	88.68
12	110.6	86.65
13	118.79	84.52
14	127	82.17
15	136.6	79.12
16	146.2	75.6
17	155.8	71.6
18	165.4	66.93
19	175	61.6
20	184.5	55.52
21	194	48.44
22	203.5	39.53
23	213	27.21
TIP	222.5	0

The chord distribution does not conform to an elliptic outline. How does one now retrace back to the original ellipse and figure out what exactly happened in terms of the quasi-elliptical or near elliptical wing shape that resulted? What was the reason for not conforming to a regular ellipse? Was it a shortcut to save time resulting in a compromise? There are apparently no records to investigate this theory, however we do have excellent evidence; the resulting wing shape which we



can scrutinise and perhaps figure out how it was derived.



One of the assumptions here is that the original ellipse was 90 by 450 inches, and it will be seen later that this assumption is somewhat justified only by the tip radius and that any elliptical ratio could have been used.

The tabulated airfoil coordinates in Drawing No. 300 08 SHT 8 must have been made at an early stage conforming to a particular ellipse as well as a predefined airfoil thickness distribution. The airfoil thickness distribution will be looked at later as to how exactly that was determined. Later, when wing geometry changes were required, redoing the coordinates was time prohibitive as these were computed semi-graphically. It was more practical to use already tabulated loft points by scaling and repositioning as the design changed.

The approach of solving this problem was somewhat intuitive, so as a first step it was decided to shift the chords in Figure 6 with their positions and lengths satisfying an elliptic shape. This is calculated as per Table 2 and shown in Figure 7.

While the chords were kept the same, the semi-span column in Table 2 was calculated from Equation 1 as follows:

$$y_s = \frac{b}{2}\sqrt{1-\left(\frac{C}{C_0}\right)^2} \qquad \textit{Equation 4}$$

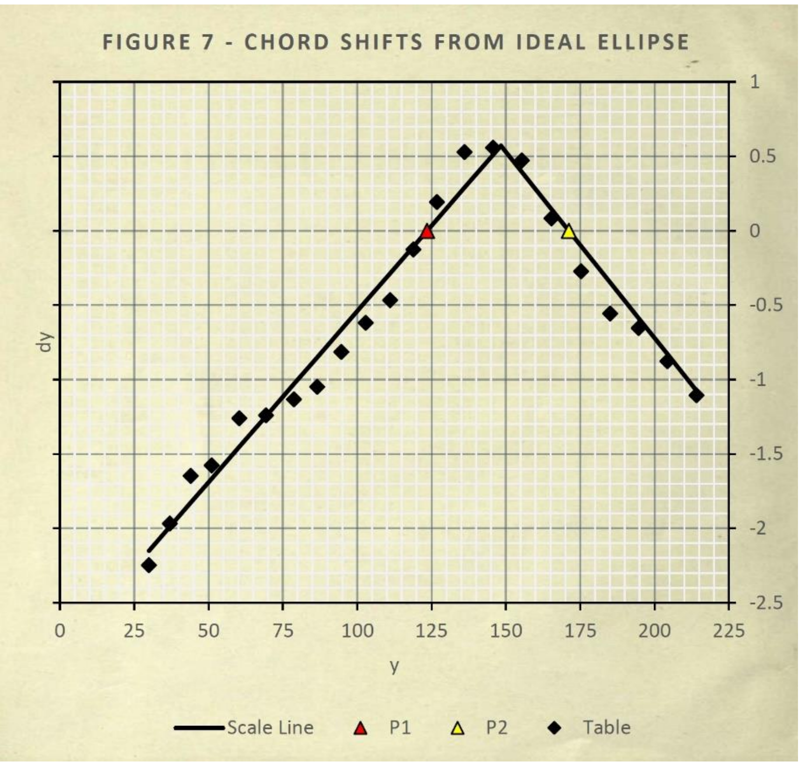
TABLE 2 Elliptic Chord Distribution				
Station	y in.	y_s in.	$y - y_s$ in.	Chord in.
0	0	2.84	-	100
2	27.7	29.95	-2.25	99.09
3	35	36.97	-1.97	98.61
4	42.3	43.95	-1.65	98.03
5	49.4	50.98	-1.58	97.34
6	59.1	60.36	-1.26	96.25
7	68.1	69.34	-1.24	95.02
8	77.6	78.73	-1.13	93.53
9	85.47	86.52	-1.05	92.13
10	93.84	94.65	-0.81	90.5
11	102.21	102.83	-0.62	88.68
12	110.6	111.07	-0.47	86.65
13	118.79	118.92	-0.13	84.52
14	127	126.81	0.19	82.17
15	136.6	136.07	0.53	79.12
16	146.2	145.64	0.56	75.6
17	155.8	155.33	0.47	71.6
18	165.4	165.32	0.08	66.93
19	175	175.27	-0.27	61.6
20	184.5	185.06	-0.56	55.52
21	194	194.65	-0.65	48.44
22	203.5	204.38	-0.88	39.53
23	213	214.10	-1.10	27.21
TIP	222.5	221.22	-	0

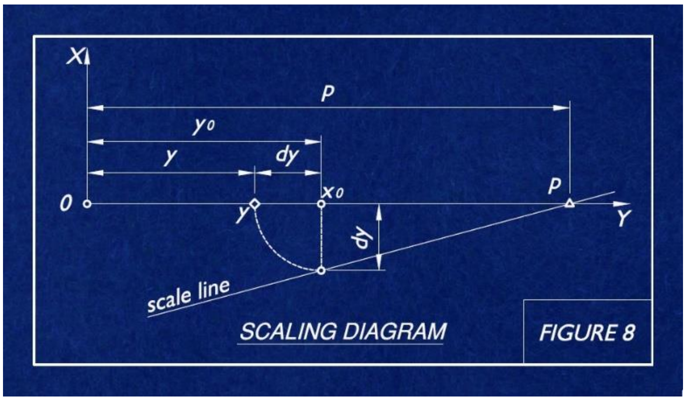
y is the semi-span, x_S is the corrected semi-span, b is the span of 445 inches, C_0 is the root chord of 100 inches, and C is the local chord.

It can be seen that the shifted distances of the chords were diminishing from the root up to station 16, then increasing towards the tip. A plot was then made of the shifts versus semi-span, Figure 7. It can be seen from the trend lines that some form of scaling is involved about two points at semi-spans of 123 and 171 inches. It also shows that two sets of chord stations have been dilated/translated which immediately indicates a composite curve, involving perhaps two ellipses rather than one. This may be compared to Figure 8 where point y_0 is scaled about point p to point y. Any point y_0 may be dilated about p by measuring the vertical distance to the scale line then translating to point y horizontally by this distance.

The scaling lines can be expressed as:

$$dy_{inboard} = .023y - 2.84$$
$$dx_{outboard} = 4.28 - .025y \qquad \textit{Equation } 5$$





Reversing the chords from their elliptical positions with the above equations and tabulating is shown in Table 3. The error in Station positions is up to ¼” and this is not the desired solution, but is shown as a step towards a solution. Also, in Figure 7, the wavy pattern around the scale lines indicates a periodicity and a more refined solution is desired.

TABLE 3 Calculated Elliptic Semispans				
STA	y in.	y_s in.	$y_{calculated}$ in.	Error in.
0	0	-2.84	0.07	0.07
2	27.7	29.95	27.80	0.10
3	35	36.97	34.98	-0.02
4	42.3	43.95	42.12	-0.18
5	49.4	50.98	49.31	-0.09
6	59.1	60.36	58.91	-0.19
7	68.1	69.34	68.09	-0.01
8	77.6	78.73	77.70	0.10
9	85.47	86.52	85.67	0.20
10	93.84	94.65	93.99	0.15
11	102.21	102.83	102.35	0.14
12	110.6	111.07	110.78	0.18
13	118.79	118.92	118.81	0.02
14	127	126.81	126.88	-0.12
15	136.6	136.07	136.36	-0.24
16	146.2	145.64	146.15	-0.05
17	155.8	155.33	155.72	-0.08
18	165.4	165.32	165.46	0.06
19	175	175.27	175.17	0.17
20	184.5	185.06	184.71	0.21
21	194	194.65	194.07	0.07
22	203.5	204.38	203.55	0.05
23	213	214.10	213.03	0.03
TIP	222.5	-	-	-

At this point it can be expected that two curves are involved, these being ellipses on which rests the original supposition. The chords were treated as two sets – Inboard from STA 1 to STA 15 and Outboard from STA 16 to STA 23. Elliptic curve-fitting was used for each set with the following formulae:

$$C = C_0 \cdot \sqrt{1 - 4\left(\frac{y}{b}\right)^2} + k \qquad \textit{Equation 6}$$

$$C = C_0 \cdot \sqrt{1 - 4\left(\frac{y - d}{b}\right)^2} + k \qquad \textit{Equation 7}$$

Regression analysis yielded coefficients in Table 4.

TABLE 4 Equation 6 & 7 coefficients		
Variable	Inboard	Outboard
b	522.289	519.972
C_0	140.451	105.359
k	-40.571	2.043
d	-	-39.46

TABLE 5 Elliptic chord fit - Equations 6 & 7				
STA	y	C	C_{calc}	Res.
0	0	100	-	-
2	27.7	99.09	99.09	0.00
3	35	98.61	98.61	0.00
4	42.3	98.03	98.03	0.00
5	49.4	97.34	97.34	0.00
6	59.1	96.25	96.24	-0.01
7	68.1	95.02	95.02	0.00
8	77.6	93.53	93.54	0.01
9	85.47	92.13	92.14	0.01
10	93.84	90.5	90.50	0.00
11	102.21	88.68	88.68	0.00
12	110.6	86.65	86.66	0.01
13	118.79	84.52	84.51	-0.01
14	127	82.17	82.15	-0.02
15	136.6	79.12	79.13	0.01
16	146.2	75.6	75.6	-
17	155.8	71.6	71.61	0.01
18	165.4	66.93	66.91	-0.02
19	175	61.6	61.60	0.00
20	184.5	55.52	55.55	0.03
21	194	48.44	48.41	-0.03
22	203.5	39.53	39.54	0.01
23	213	27.21	27.21	0.00
TIP	222.5	0	-	-

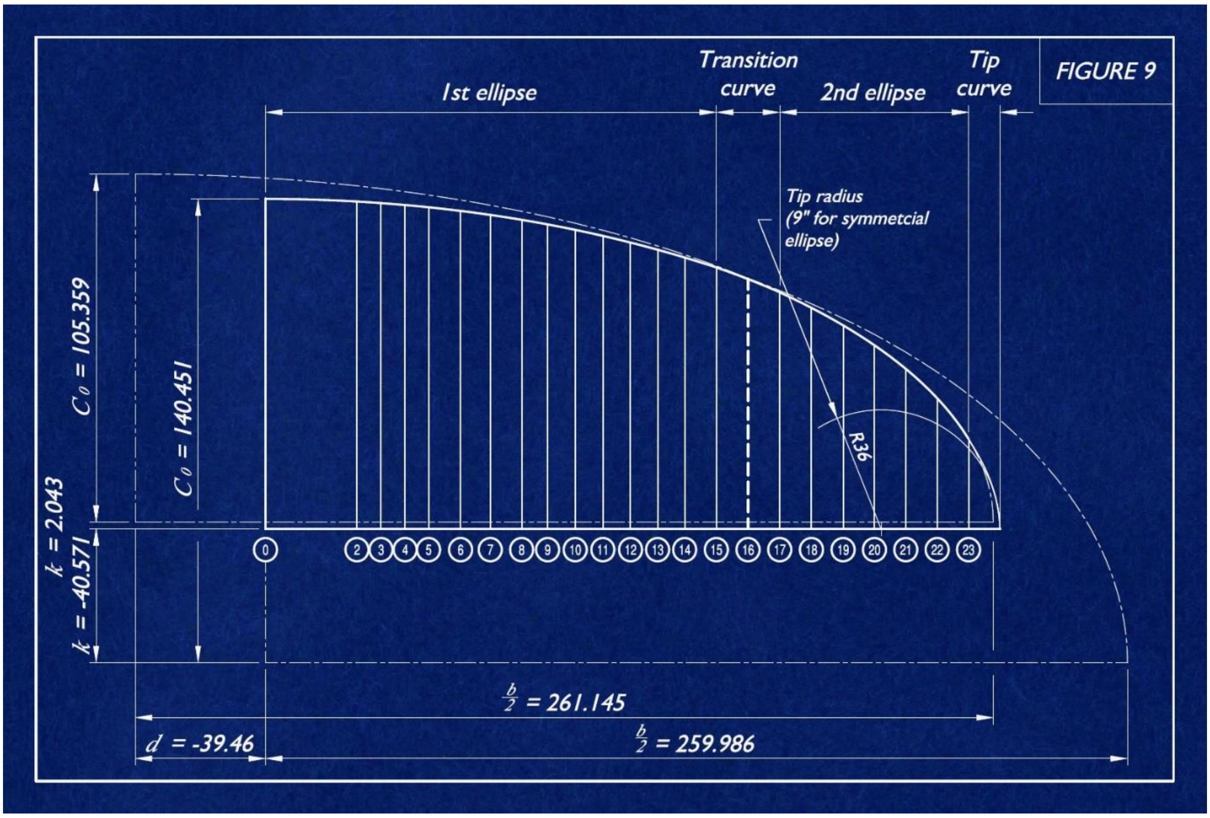


FIGURE 9

Station 16 chord is assumed to have been originally interpolated and therefore left unaltered. The result is shown in Table 5 and Figure 9.

Table 5 shows an excellent fit. The inboard error is generally one hundredths of an inch. The outboard portion error is up to .03 inches, this can be attributed to errors in rounding off and translating relatively small chord dimensions. It must be remembered that many calculations may have been done graphically in 1934 to save time.

In Figure 9, the two ellipses have an excellent fit to the chords, and STA 16 is shown dotted. It may have been interpolated for a smooth transition between the two ellipses. Also shown, is the 9 inch tip radius (Supermarine Drawing No. 300 08 Sheet 1) which has been blended by a curve segment from STA 23. More about the wing tip later on.

The next step is to figure out a common ellipse for both sets of chords. One approach would be to consider that each set of chords may be scaled as a unit. The scaling may be un-proportional in the x and y axes. Each set may then be truncated, extended, and positioned to best fit a chosen ellipse.

Curve fitting for the two sets of chords to an ellipse of 1 to 2.5 ratio was done with Equation 7. The root chord of 90 inches and at this ratio gives a semi-span of 225 inches.

$$C = C_{0r} \cdot \sqrt{1-4\left(\frac{y_0}{b_0}\right)^2} \qquad \textit{Equation 8}$$

y_0 and b_0 are the root chord and semi-span of the prototype ellipse, i.e. 90 and 225 inches respectively, and were derived from Equations 8 and 9. C_{0r} is the root chord of the prototype ellipse.

$$y = m_y \cdot y_0 + k_y \qquad \textit{Equation 9}$$

$$C = m_c \cdot C_0 + k_c \qquad \textit{Equation 10}$$

m, with suffixes y and c is a scale factor for the y-axis and chords respectively, and k is a constant for curve fitting flexibility.

The final curve fitting equation is derived by substituting Equations 9 and 10 into Equation 8.

$$C = k_c + m_c \cdot C_{0r} \cdot \sqrt{1 - 4\left(\frac{k_y - y}{b_0 \cdot m_y}\right)^2} \qquad \textit{Equation 11}$$

Regression analysis yielded coefficients are shown in Table 6.

TABLE 6
Equation 11 Coefficients

Variable	Inboard	Outboard
k_y	-0.398	-39.46
k_c	-35.965	2.043
m_y	1.14582	1.15549
m_c	1.50963	1.17065

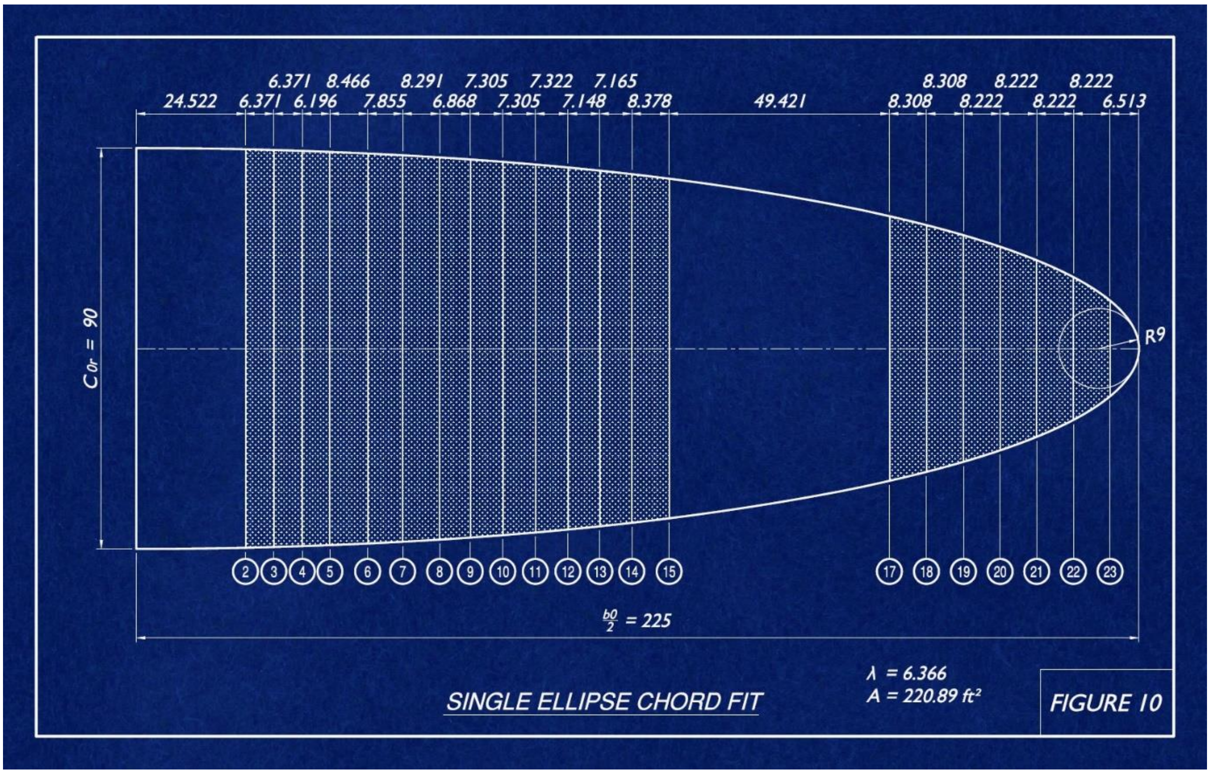


TABLE 7 Single Ellipse Chord Fit					
y	*C*	C_{calc}	*Res.*	y_0	C_0
0	100	-	-	0.00	90.00
27.7	99.09	99.09	0.00	24.52	89.46
35	98.61	98.61	0.00	30.89	89.15
42.3	98.03	98.03	0.00	37.26	88.76
49.4	97.34	97.34	0.00	43.46	88.31
59.1	96.25	96.23	-0.02	51.93	87.57
68.1	95.02	95.02	0.00	59.78	86.77
77.6	93.53	93.53	0.00	68.07	85.78
85.47	92.13	92.14	0.01	74.94	84.86
93.84	90.5	90.50	0.00	82.25	83.77
102.21	88.68	88.68	0.00	89.55	82.56
110.6	86.65	86.66	0.01	96.87	81.23
118.79	84.52	84.51	-0.01	104.02	79.80
127	82.17	82.15	-0.02	111.19	78.24
136.6	79.12	79.13	0.01	119.56	76.24
146.2	75.6	75.6	-	127.94	73.90
155.8	71.6	71.61	0.01	168.98	59.42
165.4	66.93	66.91	-0.02	177.29	55.41
175	61.6	61.60	0.00	185.60	50.88
184.5	55.52	55.55	0.03	193.82	45.71
194	48.44	48.41	-0.03	202.04	39.60
203.5	39.53	39.54	0.01	210.27	32.03
213	27.21	27.21	0.00	218.49	21.50
222.5	0	-	-	225.00	0

The residual column in Table 7 shows an excellent fit, comparable to Table 5.

Before the decision to go with an elliptic wing in Mid November 1934, the first ellipse for revising type 224 was laid out at least six weeks earlier. This was an option decided upon as part of the private venture between Vickers and Supermarine. It is reasonable to assume that the parameters for the elliptical wing were derived from the modified F.7/30 specification as per drawing No. 300 00 Sheet 2. The published 3 view drawing had a scale bar which was used to measure the wing geometry with the following results:

Wing span: 35 ft 4 in.

Root Chord: 90 in.

Tip Chord: 36 in.

Dihedral: 4 deg.

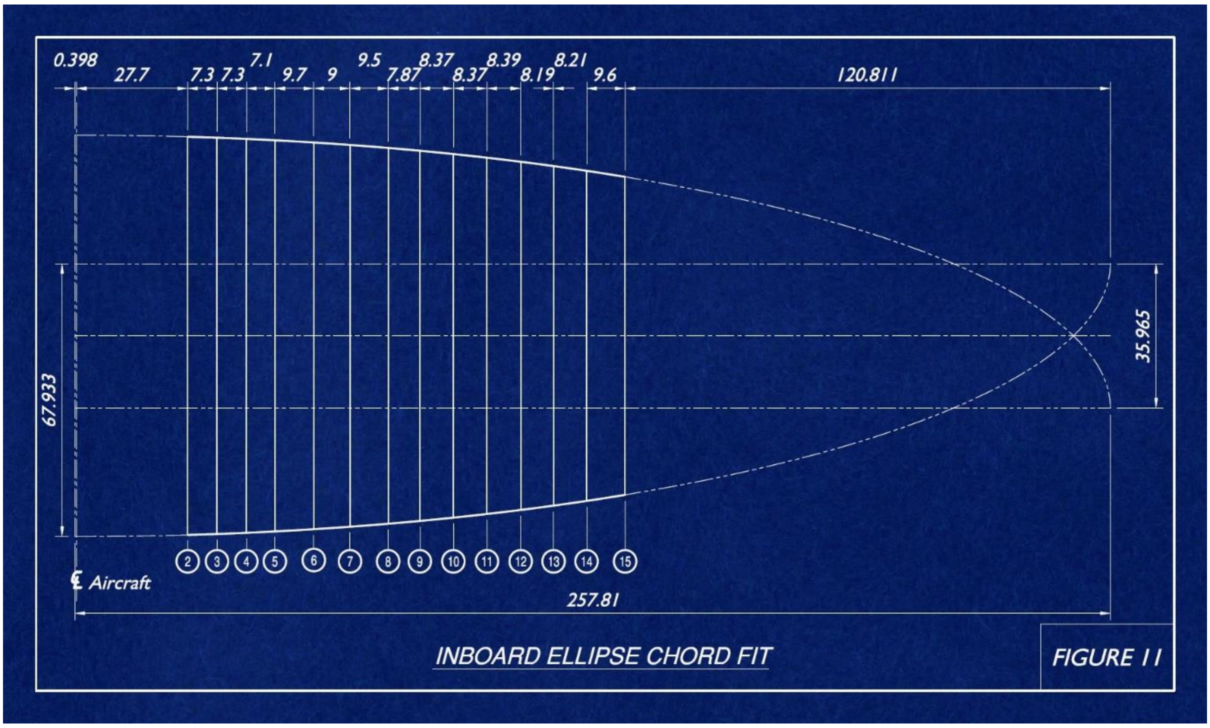
Taper ratio: 1 : 2.5

Area: 184 sq. ft

Figure 9 shows a 90 in. root chord ellipse with a 1:2.5 span ratio, these being sensible round figures to start with as the number of guns were increasing from 4 to 6 and later to 8.

Ultimately, 8 guns were anticipated and the planform had to be increased in size. Exactly how this was done can only be speculated but it is quite probable that this was Drawing Office work, rather than a result of aerodynamic calculations. The shaded areas in Figure 9 are sections used to shape the final wing. A possible scenario of manipulating the two wing sections to obtain the final chord distribution may have been as follows:

1. Inboard section
    a. Chords were scaled by a factor of by a factor of 1.50963, then shortened by 35.965 inches.
    b. Distances between chords were scaled by a factor of 1.14582.
    c. The chords were then positioned so that Station 2 was 27.7 in. from aircraft centreline. Or, alternatively, the 25.43 in. distance of Station 2 was scaled by 1.14582, then shifted by .398 inches towards aircraft centreline.

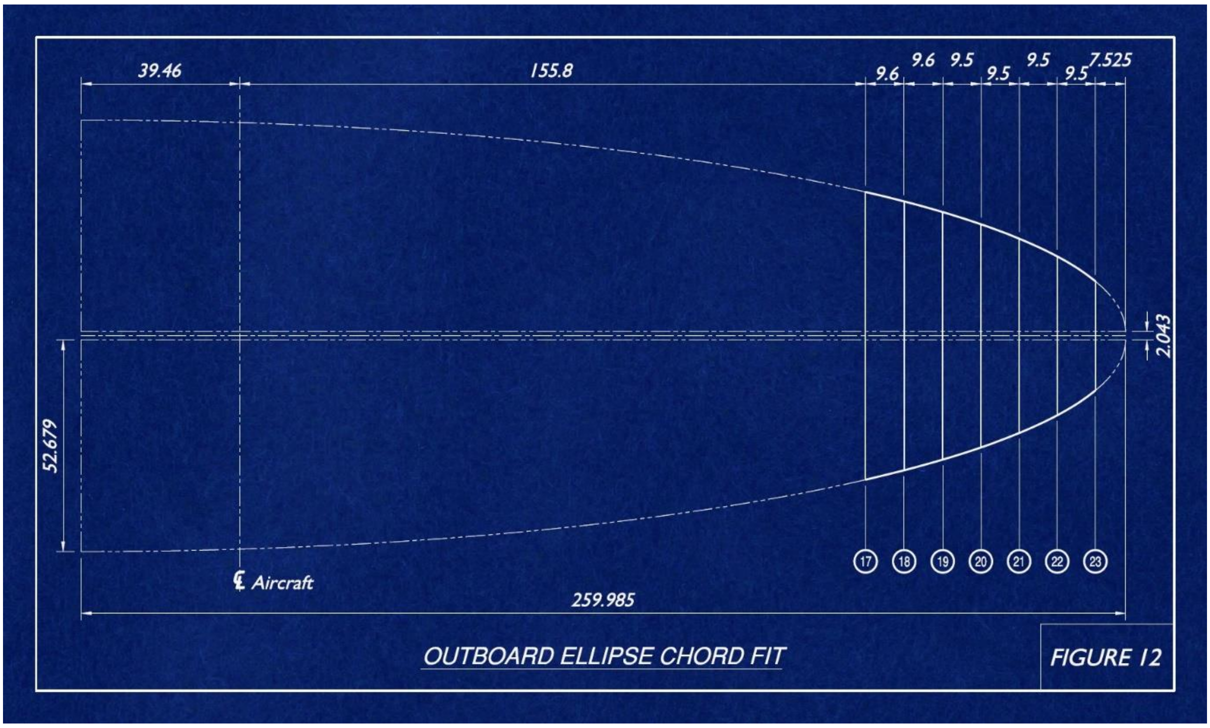


INBOARD ELLIPSE CHORD FIT
FIGURE 11

2. Outboard section
    a. Chords were scaled by a factor of by a factor of 1.17065, then lengthened by 2.043 inches.
    b. Distances between chords were scaled by a factor of 1.15549.
    c. The chords were then positioned so that Station 17 was 155.8 in. from aircraft centreline. Or, alternatively, the 168.58 in. distance of Station 17 was scaled by 1.15549, then shifted by 39.46inches towards aircraft centreline.

The inboard and outboard scaling results are shown in Figures 11 & 12, and the combined chords in Figure 13. The gap of 18.2 inches was divided with the interpolated chord of 76.1 inches as Station 16.

The root chord was then affirmed as 100 inches, while the tip positioned at 222.5 inches or 9.5 inches outboard from the last chord (Station 23). This arrangement very nearly approximated an ellipse. This was crucial or the result would have been an aerodynamicist's nightmare in 1934. As for the tip radius, due to scaling, extending and translating, the original 9 inches radius was perhaps deemed as appropriate as any other guess, and besides, time was on the design team's side.



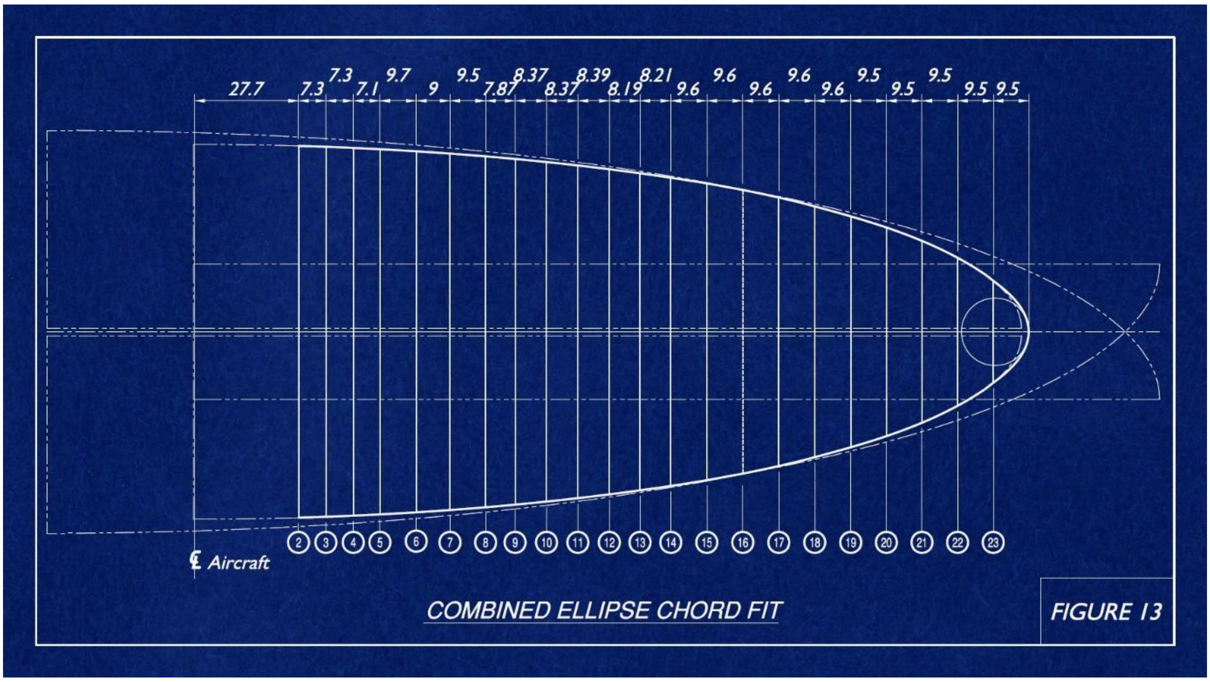
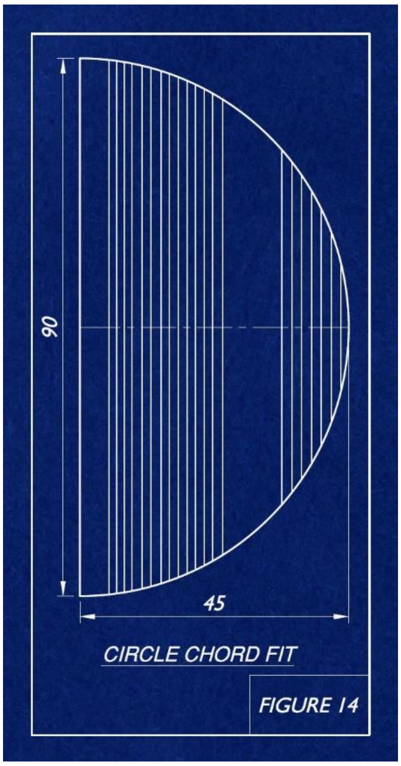


Figure 13 is practically a reflection of Figure 9 except for the inboard ellipse which has been placed at the origin (c/l of aircraft) in Figure 8 and yielded accurate chord lengths to within two decimal places. Also, the chords are placed centrally in the later.

This just shows than any ellipse could have been used to fit the chords. An extreme case would be a circle as shown in Figure 14 and analogous to Figure 10.



CIRCLE CHORD FIT

FIGURE 14

Leading Edge Shape

The chord distribution may well have been done numerically as well as graphically, and hence discrepancies. Of course, some chords, especially at the tip end, may have been shifted further, either numerically or graphically, as discrepancies up to 3 hundredths of an inch show.

The leading edge curve, on the other hand, may have been eyeballed by the drawing office. During frequent discussions between the design team members a considerable amount of sketches were made particularly by Mitchell. The change to a heavier Merlin engine required a forward location of the wing aerodynamic centre and the 'D' wing leading edge together with the spar had to have sufficient chord-wise depth for adequate strength among other things.

For the purpose of studying the curve and its nature – (whether this nature was intended or coincidental form Supermarine's part) a simple, cubic relation will be used here.

It should be pointed out here that the leading edge shape distorts the planform in Fig. 13, however the elliptical chord distribution remains unaltered. The spar is located at 24.5% from root chord leading edge or 24.5 inches. The wing planform axis, however, coincides with the centre of 9 inch tip radius. This is at 35.5% or 35.5 inches from root chord leading edge.

An elliptic planform could be drawn with two semi ellipses about the wing axis as shown in Figure 15. The tip limiting radii of the ellipses are 5.66 and 18.7 inches respectively.

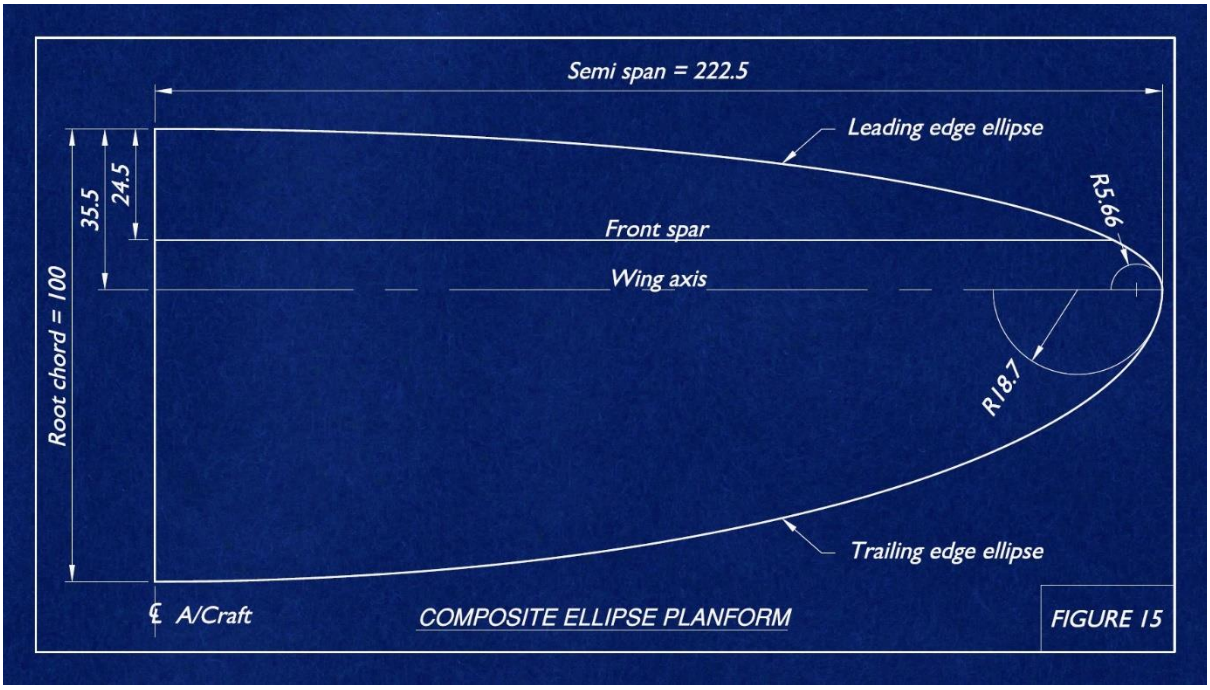
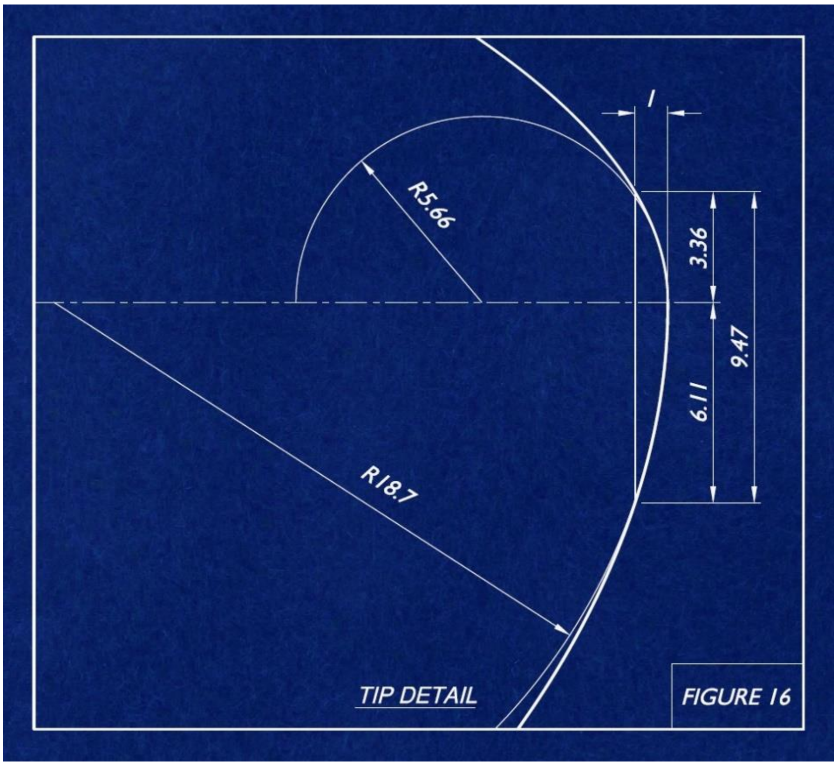


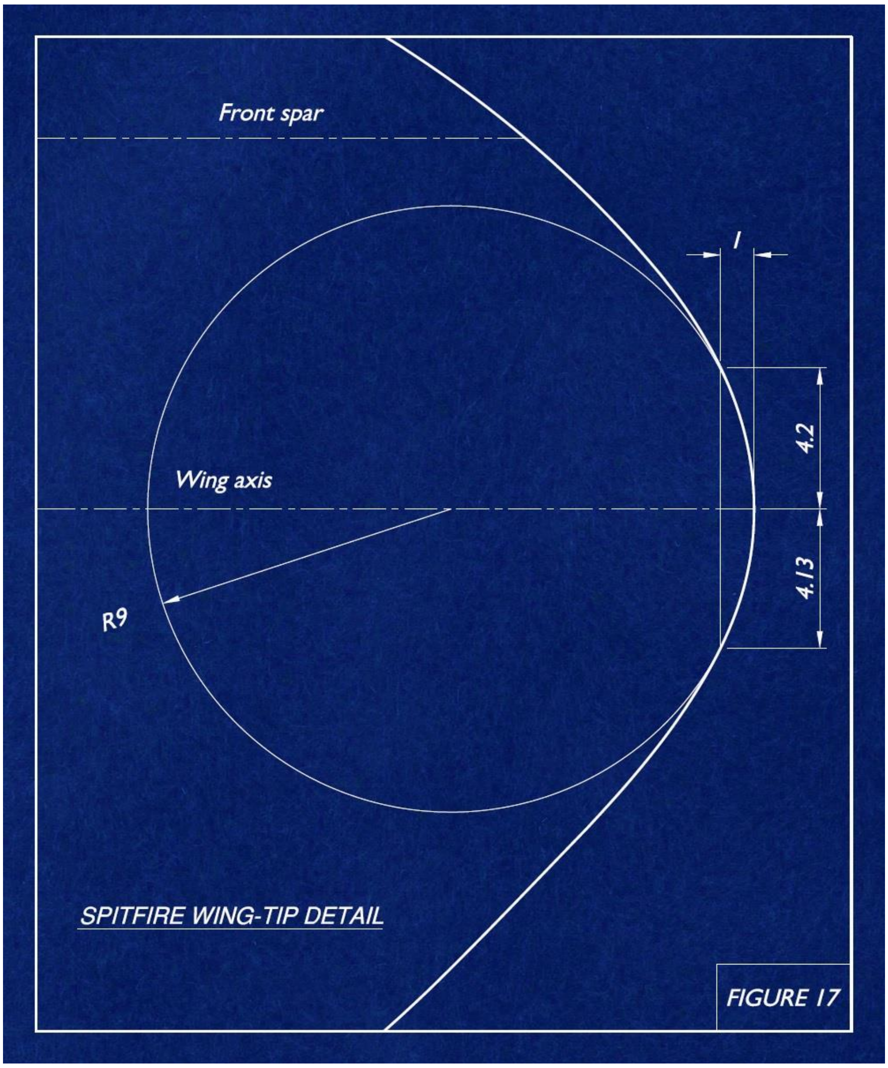
Figure 16 shows a zoomed view of the tip and a chord of 9.47 inches at STA 221.5 (1 inch from the tip). The wing axis splits the chord at 35.5% or 3.36 inches from the leading edge, identically as at root chord. This gives the tip an unsymmetrical look.

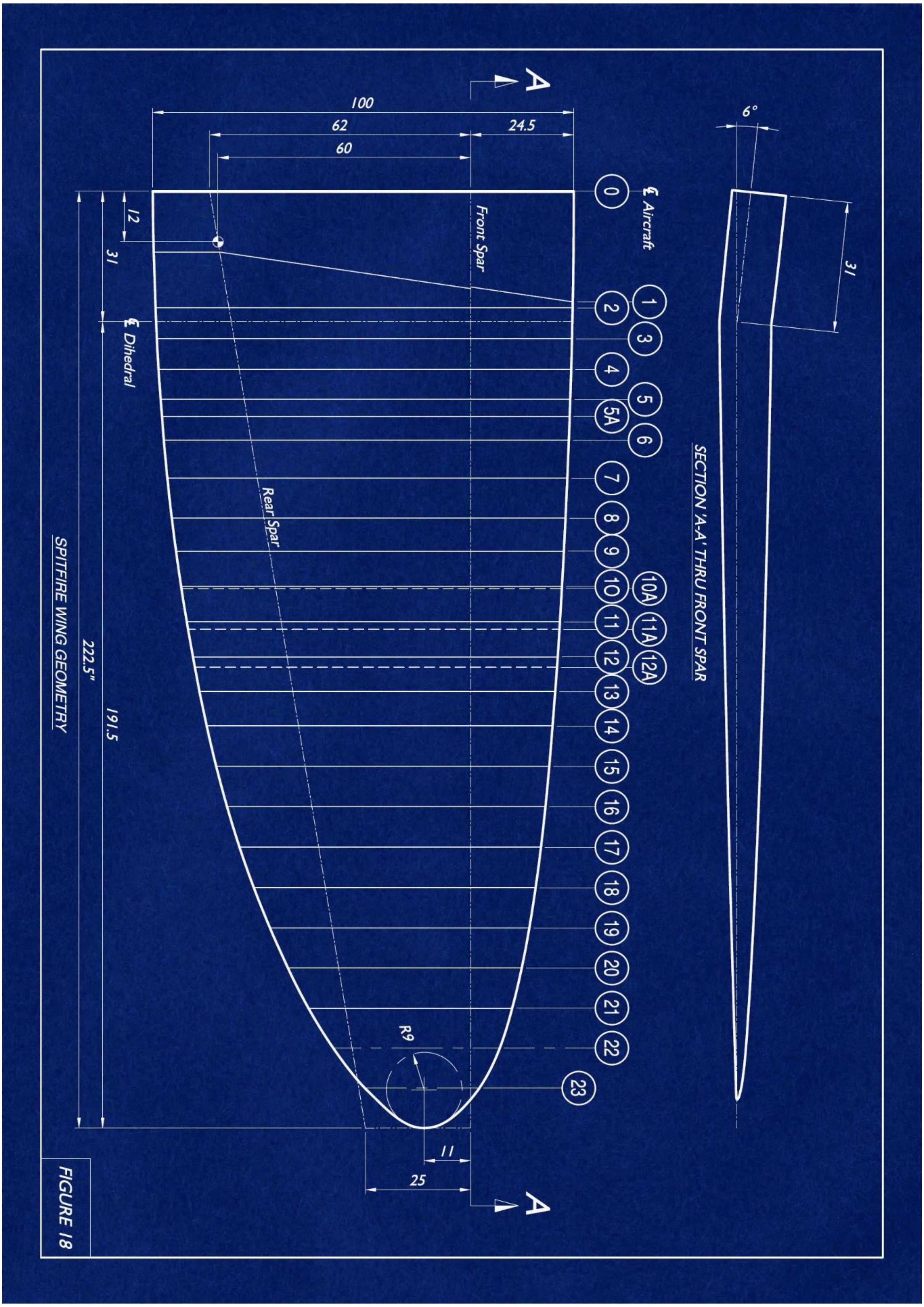
The beauty and elegance of the Spitfire wing is in its symmetrical wing tip with a single 9 inch radius (see Figure 18). The limiting ratio of axis position is therefore 50% at the tip. This can be seen in on the Spitfire wingtip, Figure 17, where the chords tend to be placed at their 50% length as they approach the tip.

The chords are positioned about the wing axis is such a way that this ratio changes from 35.5% to 50% by a certain rule. For the Spitfire wing this can be plotted using leading edge offsets 'A' as per Supermarine drawing no. 337 08 SHT 8.



1
R5.66
3.36
9.47
6.11
R18.7
TIP DETAIL
FIGURE 16


Front spar
1
4.2
Wing axis
4.13
R9
SPITFIRE WING-TIP DETAIL
FIGURE 17



SPITFIRE WING GEOMETRY
FIGURE 18
SECTION 'A-A' THRU FRONT SPAR
A
A
100
62
24.5
60
12
31
222.5"
191.5
℄ Aircraft
℄ Dihedral
Front Spar
Rear Spar
R9
11
25
6°
31
0
1
2
3
4
5
5A
6
7
8
9
10
10A
11
11A
12
12A
13
14
15
16
17
18
19
20
21
22
23

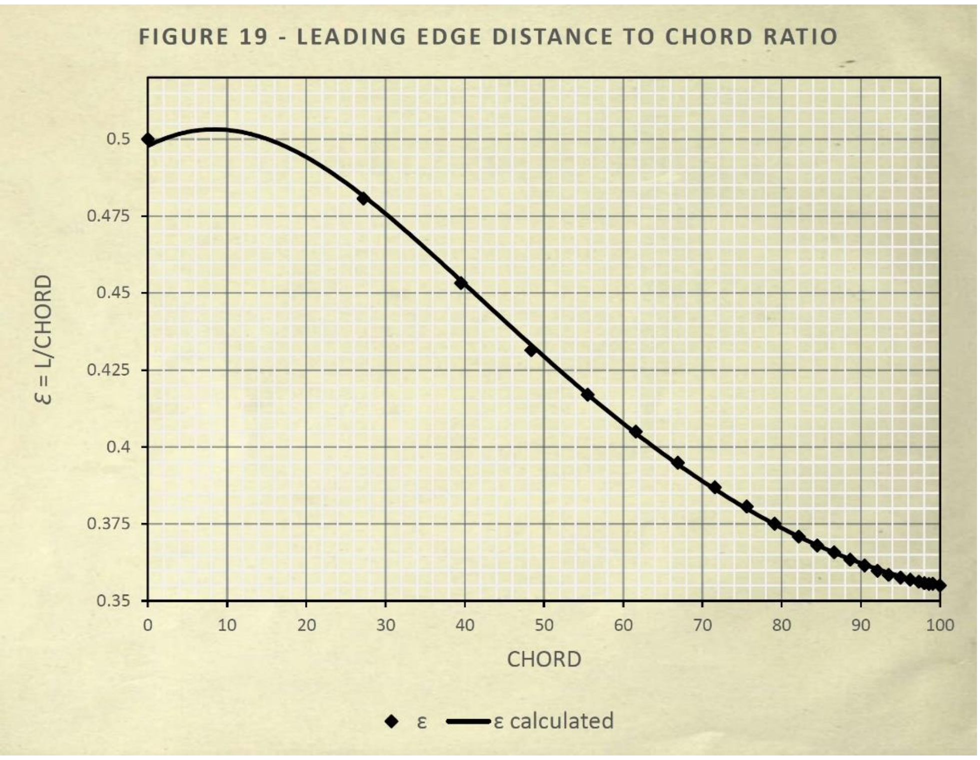


Figure 19 shows the Leading Edge 'L' distance to Chord ratio versus Chord as the Chord varies along

TABLE 8 Leading edge distance from neutral axis L to Chord ratio					
Station	Semispan	Chord	L	$\varepsilon = \frac{L}{Chord}$	ε
	in.	in.	in.		Calculated
0	0	100	35.5	0.355	0.3548
2	27.7	99.09	35.24	0.3556	0.3553
3	35	98.61	35.06	0.3555	0.3556
4	42.3	98.03	34.88	0.3558	0.3559
5	49.4	97.34	34.68	0.3563	0.3564
6	59.1	96.25	34.35	0.3569	0.3571
7	68.1	95.02	33.98	0.3576	0.358
8	77.6	93.53	33.53	0.3585	0.3592
9	85.47	92.13	33.15	0.3598	0.3603
10	93.84	90.5	32.72	0.3615	0.3618
11	102.21	88.68	32.23	0.3634	0.3636
12	110.6	86.65	31.7	0.3658	0.3657
13	118.79	84.52	31.1	0.368	0.3681
14	127	82.17	30.48	0.3709	0.3709
15	136.6	79.12	29.68	0.3751	0.3749
16	146.2	75.6	28.78	0.3807	0.38
17	155.8	71.6	27.7	0.3869	0.3863
18	165.4	66.93	26.43	0.3949	0.3943
19	175	61.6	24.95	0.405	0.4045
20	184.5	55.52	23.15	0.417	0.4171
21	194	48.44	20.9	0.4315	0.433
22	203.5	39.53	17.92	0.4533	0.454
23	213	27.21	13.08	0.4807	0.4816
TIP	222.5	0	0	0.5	0.4978

the semi span. Offsets 'A' are from the front spar and are renamed 'L' as leading edge distance from wing axis ('A' + 11 inches) and the tabulated values are shown in Table 8.

The 'S' curve fit was made with the following cubic relation:

$$\left(\frac{1}{\varepsilon}\right)^2 = a_0 + a_1 \cdot C + a_2 \cdot C^2 + a_3 \cdot C^3 \qquad \textit{Equation 12}$$

With the coefficients shown in Table 9.

TABLE 9 Equation 12 Coefficients	
a_0	4.03566
a_1	-0.02105
a_2	0.001351
a_3	-7.495E-06

The leading edge shape factor ε is over-bulging near the tip and this is evident in Figures 17 and 19. Factor ε has maximum value of 0.50317 at 8.5 inches from the tip as shown in Figure 19. Figure 17 shows a chord of 8.33" that is split as 4.2" forward and 4.13" aft of the wing axis. For an asymptotic curve this should be, say, 4.16" and 4.17" or the two figures being nearly equal with the aft portion slightly longer.

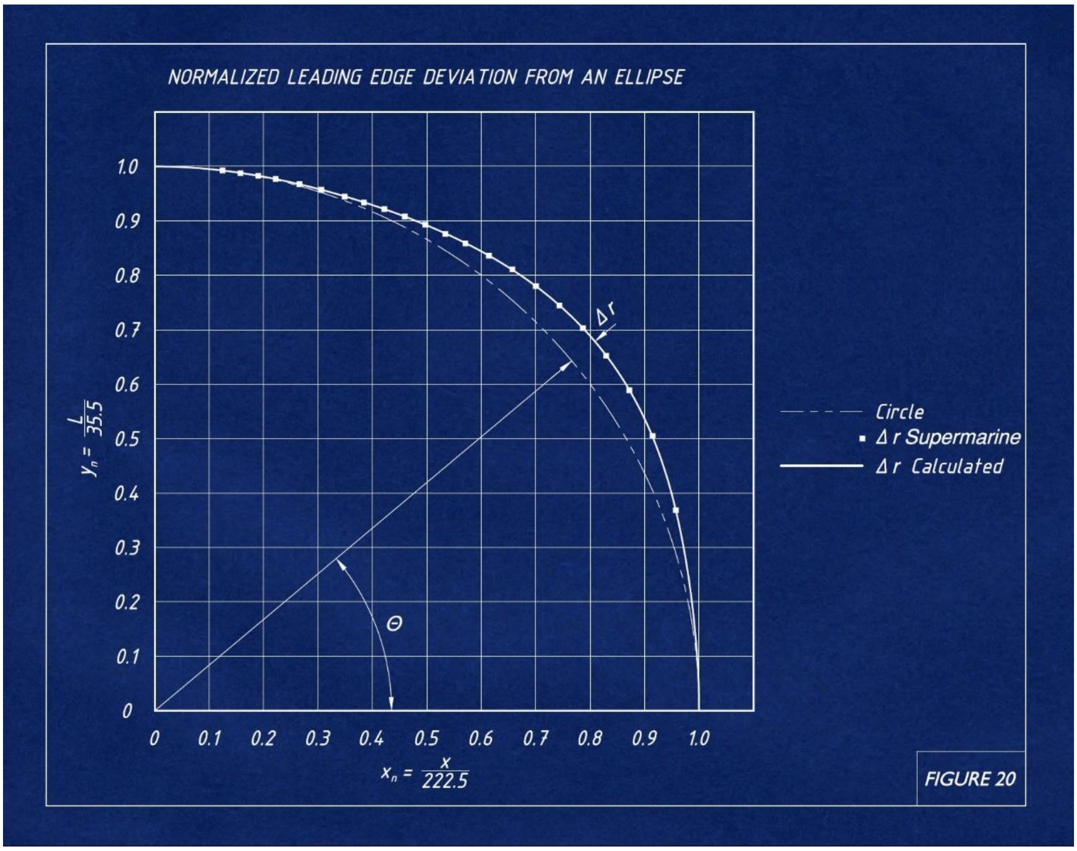
Equation 12 gives an excellent fit so that the calculated leading edge deviates maximum 1/16" at Stations 16 and 21, the average deviation being zero.

The curve in Figure 19 can be deemed to be the control for the leading edge shape. The nature or character of this curve is to gradually distort or morph the planform along its span. Instead of an 'S' shape this curve could have a different shape or it could even be a straight line. By varying this curve a wing planform may be morphed for example, from an asymmetric root to a symmetric tip, or vice versa, either way resulting in an aesthetic look. The Spitfire designers, perhaps unwittingly, achieved this effect even with an over-bulge at the tip. It will be shown that a straight line will just as well result in an aesthetic effect. Factors other than aesthetics certainly had a role in shaping the leading edge, such as the position of the mean aerodynamic chord, however, it can be shown that its position does not have to alter with the bulge removed. More about the various effects the curve can have on wing planforms this later on.

Previous work on the leading edge shape

An excellent curve fit (1 to 2 hundredths of an inch) was given in "The Spitfire Wing – A Mathematical Model" by this writer. For the sake of completeness, the derivation method will be given here in a simplified but complete form.

The leading edge points were first normalized (where root chord and span had a value of unity). A circular arc was then superimposed and the value of 1 was then subtracted from the radial distances of the leading edge points. This is shown in Figure 20. A probability curve called Beta Distribution (probability distribution) was iteratively fitted to these differences, Δr with respect to θ. This is shown in Figure 21.



The equation given in "The Spitfire Wing – A Mathematical Model" is somewhat cumbersome. After substitution and simplification it reduces as follows:

$$\Delta r = 44.5 \cdot \left(0.573 \cdot \theta + 0.05\right)^{4.33} \cdot \left(-0.573 \cdot \theta + 0.95\right)^{5.4} \qquad \textit{Equation } 13$$

where

$$\theta = a\tan\left(\frac{y_n}{x_n}\right) = a\tan\left(\frac{445 \cdot L}{71 \cdot x}\right) \qquad \textit{Equation} 14$$

$$y_n = \frac{L}{35.5} \qquad \textit{Equation } 15$$

$$x_n = \frac{x}{222.5} \qquad \textit{Equation } 16$$

The leading edge curve may now be expressed in parametric form:

$$x = 222.5 \cdot \left(1 + \Delta r\right) \cdot \cos\left(\theta\right) \qquad \textit{Equation } 17$$

$$L = 35.5 \cdot \left(1 + \Delta r\right) \cdot \sin\left(\theta\right) \qquad \textit{Equation } 18$$

The leading edge calculations are presented in Table 10.

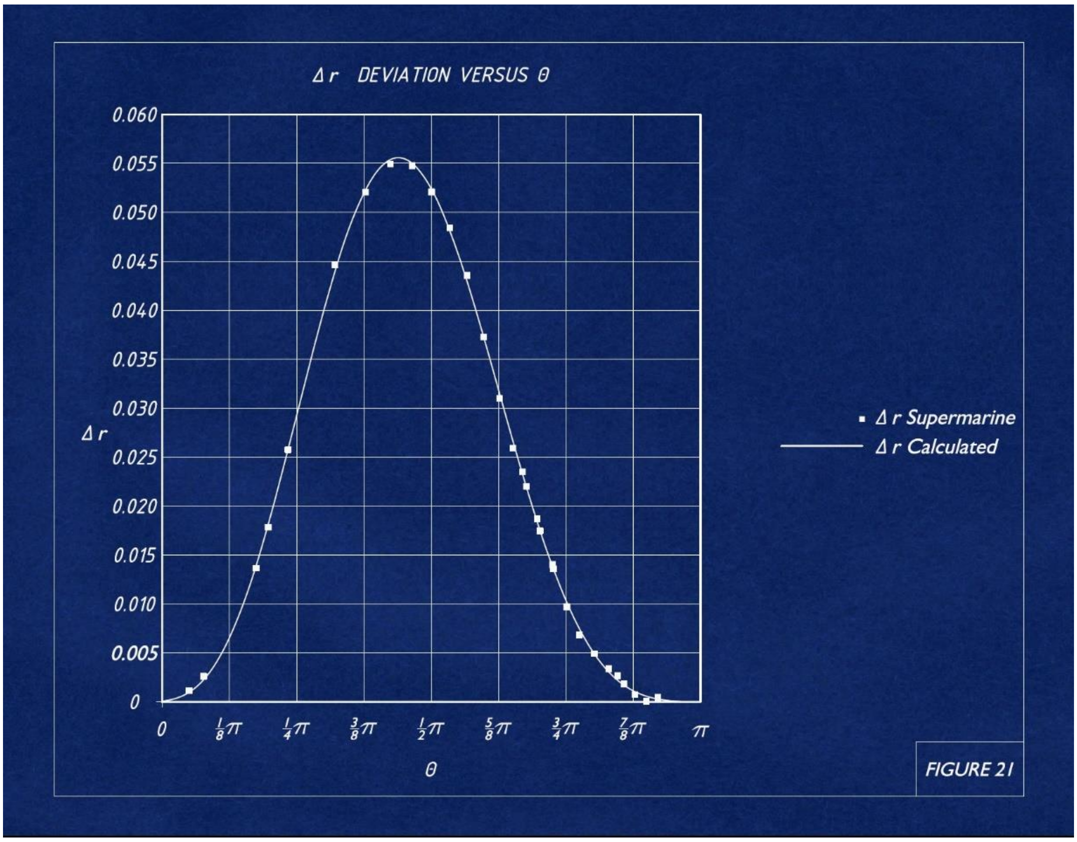


FIGURE 21

TABLE 10 - LEADING EDGE SHAPE											
ORIGINAL LEADING EDGE			NORMALIZED		$atan(y_n/x_n)$	$y_n/sin(\theta)-1$	CALCULATED VALUES				
STA	x	L	x_n=x/222.5	y_n=L/35.5	θ	Δr	Δr_{calc}	x_{calc}	L_{calc}	$error_x$	$error_L$
0	0	35.5	0	1	1.570796	0.0000	0.0000	0	35.50	0	0.00
2	27.7	35.24	0.124494	0.992676	1.446035	0.0005	0.0003	27.70	35.23	0.00	-0.01
3	35	35.06	0.157303	0.987606	1.412846	0.0001	0.0006	35.02	35.08	0.02	0.02
4	42.3	34.88	0.190112	0.982535	1.379667	0.0008	0.0010	42.31	34.89	0.01	0.01
5	49.4	34.68	0.222022	0.976901	1.347320	0.0018	0.0017	49.39	34.68	-0.01	0.00
6	59.1	34.35	0.265618	0.967606	1.302885	0.0034	0.0031	59.08	34.34	-0.02	-0.01
7	68.1	33.98	0.306067	0.957183	1.261312	0.0049	0.0049	68.10	33.98	0.00	0.00
8	77.6	33.53	0.348764	0.944507	1.217072	0.0068	0.0075	77.65	33.55	0.05	0.02
9	85.47	33.15	0.384135	0.933803	1.180530	0.0097	0.0102	85.51	33.16	0.04	0.01
10	93.84	32.72	0.421753	0.921690	1.141652	0.0136	0.0136	93.84	32.72	0.00	0.00
11	102.21	32.23	0.459371	0.907887	1.102378	0.0175	0.0175	102.21	32.23	0.00	0.00
12	110.6	31.7	0.497079	0.892958	1.062850	0.0220	0.0219	110.59	31.70	-0.01	0.00
13	118.79	31.1	0.533888	0.876056	1.023478	0.0259	0.0266	118.87	31.12	0.08	0.02
14	127	30.48	0.570787	0.858592	0.984091	0.0310	0.0316	127.07	30.50	0.07	0.02
15	136.6	29.68	0.613933	0.836056	0.937406	0.0373	0.0374	136.61	29.68	0.01	0.00
16	146.2	28.78	0.657079	0.810704	0.889684	0.0435	0.0430	146.12	28.76	-0.08	-0.02
17	155.8	27.7	0.700225	0.780282	0.839420	0.0484	0.0481	155.76	27.69	-0.04	-0.01
18	165.4	26.43	0.743371	0.744507	0.786162	0.0521	0.0523	165.44	26.44	0.04	0.01
19	175	24.95	0.786517	0.702817	0.729258	0.0548	0.0550	175.04	24.96	0.04	0.01
20	184.5	23.15	0.829213	0.652113	0.666407	0.0549	0.0554	184.58	23.16	0.08	0.01
21	194	20.9	0.871910	0.588732	0.593902	0.0521	0.0523	194.05	20.91	0.05	0.01
22	203.50	17.92	0.914607	0.504789	0.504315	0.0447	0.0440	203.37	17.91	-0.13	-0.01
23	213.00	13.08	0.957303	0.368451	0.367408	0.0258	0.0258	213.00	13.08	0.00	0.00
TIP	222.50	0	1	0	0	0.0000	0.0001	222.52	0.00	0.02	0

AIRFOIL THICKNESS DISTRIBUTION

Rather than using brutal force to fit an equation to the wing thickness distribution as was done in “The Spitfire Wing – A mathematical Model”, the writer recollects previous experience with thickness distribution on helicopter blades. The blade tapered linearly both in planform and thickness, however the thickness ratio of local maximum thickness to chord was not a straight line. This curve was tried on the Spitfire wing and gave an excellent fit as will be now shown.

Airfoil thickness distribution for the Supermarine Spitfire was derived directly from a virtual, trapezoidal wing. Maximum airfoil thicknesses of the virtual planform were applied to corresponding semi-spans on the elliptical planform. These thicknesses were then tabulated in percent of local chords. This virtual wing may have the same span and root chord as the actual wing. Having fixed the span and root chord, two additional parameters are required, namely planform taper and thickness taper.

Local airfoil thickness on a trapezoidal planform is defined as

$$T_x = \frac{t_x}{C_x} \qquad \text{Equation 19}$$

where

$$t_x = t_0 - x \cdot m_t \qquad \textit{Equation } 20$$

$$C_x = C_0 - x \cdot m_C \qquad \textit{Equation } 21$$

$$\mathrm{m}_t = \frac{t_0 - t_t}{b} \qquad \textit{Equation } 22$$

$$\mathrm{m}_C = \frac{C_0 - C_t}{b} \qquad \textit{Equation } 22$$

Equation 19 may therefore be written as

$$T_x = \frac{t_r - x \cdot m_t}{C_r - x \cdot m_C} \qquad \textit{Equation } 23$$

Best fit was obtained with coefficients shown in Table 11 with the root chord placed at 12 in from centreline of aircraft.

The equation may now be finalized:

$$T_x = \frac{12.98 - (x_0 - 12) \cdot 0.0467}{100 - (x_0 - 12) \cdot 0.2309} \qquad \textit{Equation } 24$$

The calculated values are tabulated along with the original wing in Table 12.

TBLE 11 Equation 23 Coefficients	
mt	0.0467
m_C	0.2309

As can be seen in Figure 22, the Root chord of 100 inches does not belong to aircraft centreline position (STA 0) but 12 inches outboard. The wing section at this station (12 inches outboard) is not physical as this area is enveloped in the wing root fillet and is the position of the rear spar attachment bolt. The 12 in outboard station may well have been the original root chord at first and later simply moved to the centreline for the elliptic configuration.

It is also interesting to note that when the virtual wing is extended to aircraft centreline the planform and thickness taper-ratios are 1:2 and 7:30 respectively. These round figures plausibly belonged to the original layout before scaling and chopping took place.

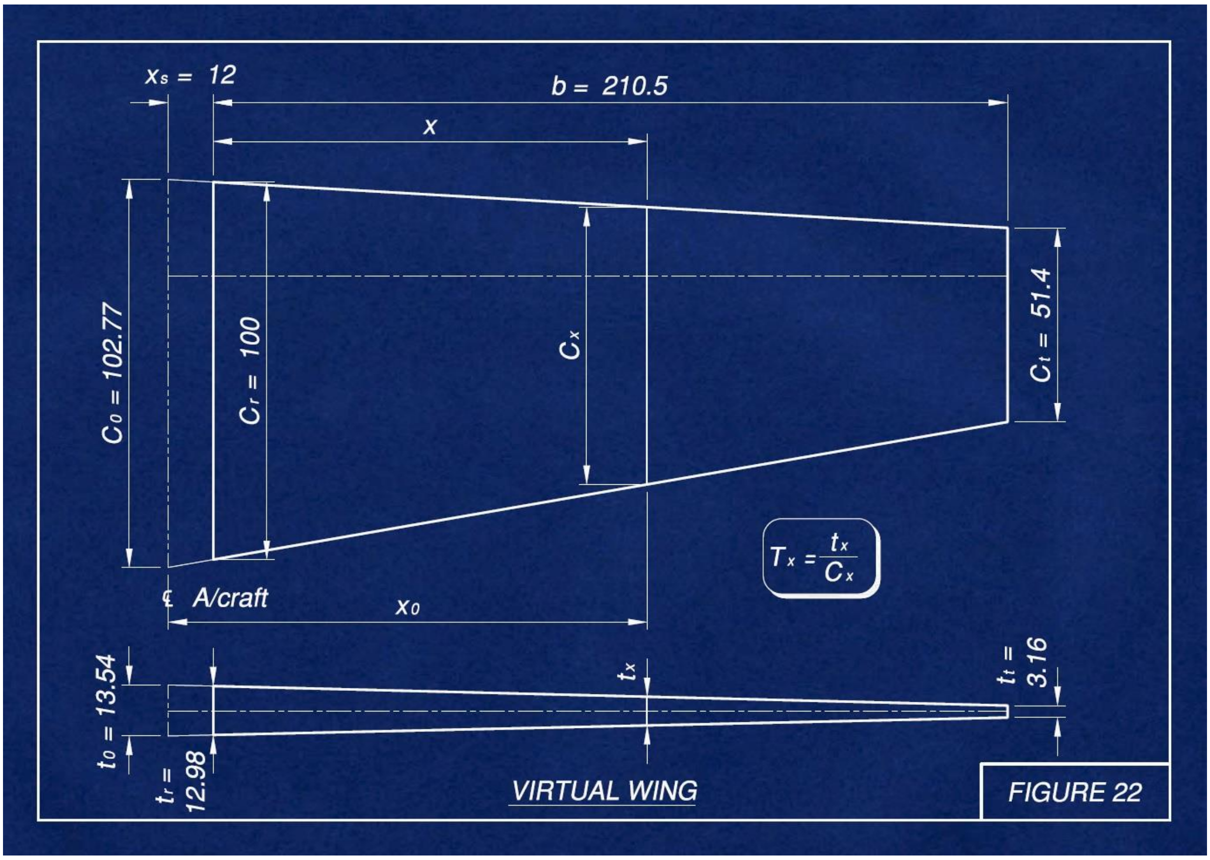
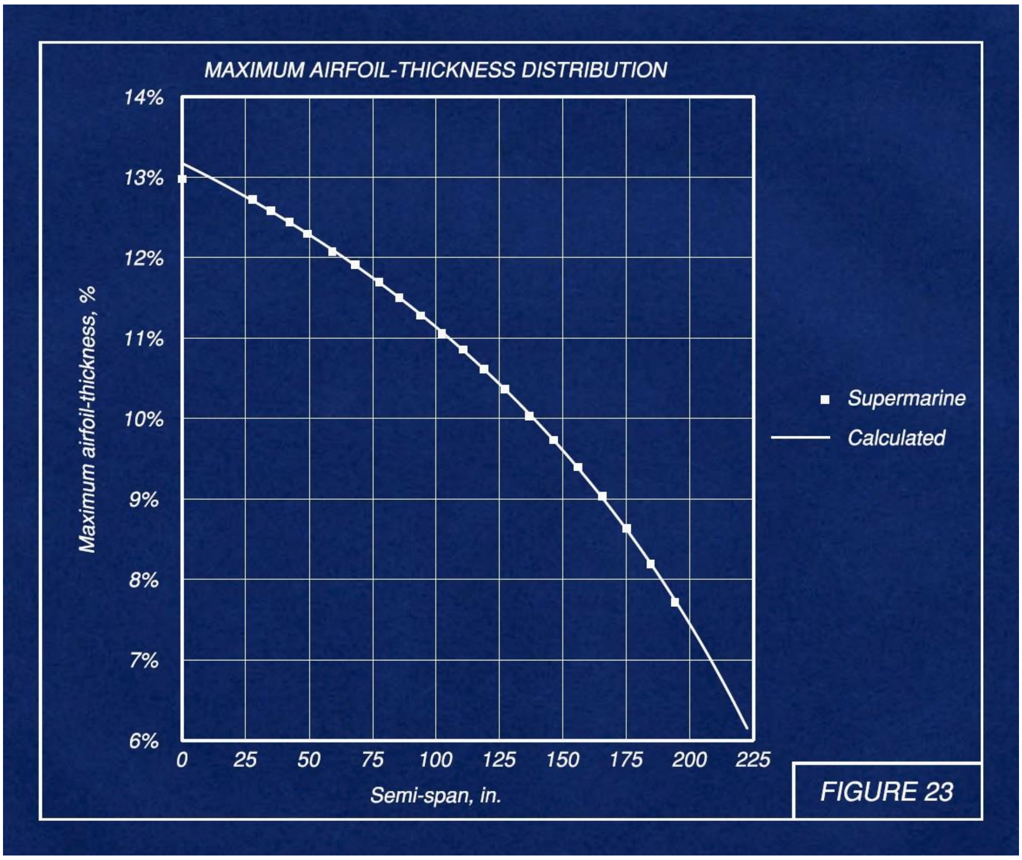


TABLE 12 Airfoil thickness distribution						
Supermarine					Calculated	
STA	X_0 [IN]	C [IN]	t_{max} [IN]	T_{max} [IN]	T_{max} [%]	error [IN]
0	0	(102.77)	-	-	13.17	-
-	(12)	(100)	12.98	12.98	12.98	0.00
2	27.7	99.09	12.61	12.73	12.71	-0.02
3	35	98.61	12.41	12.58	12.57	-0.01
4	42.3	98.03	12.2	12.45	12.44	-0.01
5	49.4	97.34	11.97	12.30	12.30	0.00
6	59.1	96.25	11.62	12.07	12.10	0.02
7	68.1	95.02	11.32	11.91	11.91	-0.01
8	77.6	93.53	10.94	11.70	11.69	-0.01
9	85.47	92.13	10.6	11.51	11.50	0.00
10	93.84	90.5	10.21	11.28	11.30	0.01
11	102.21	88.68	9.81	11.06	11.08	0.02
12	110.6	86.65	9.41	10.86	10.85	-0.01
13	118.79	84.52	8.97	10.61	10.62	0.00
14	127	82.17	8.52	10.37	10.37	0.00
15	136.6	79.12	7.94	10.04	10.06	0.02
16	146.2	75.6	7.36	9.74	9.74	0.00
17	155.8	71.6	6.73	9.40	9.39	-0.01
18	165.4	66.93	6.05	9.04	9.02	-0.01
19	175	61.6	5.32	8.64	8.62	-0.01
20	184.5	55.52	4.55	8.20	8.20	0.00
21	194	48.44	3.74	7.72	7.74	0.01
22	203.5	39.53	-	-	7.25	-
23	213	27.21	-	-	6.72	-
TIP	222.5	0	-	-	6.15	-
Virtual tapered wing						
Semi span				b	210.5	[in]
Root chord				C_r	100	[in]
Tip chord				C_t	51.40	[in]
Root airfoil thickness				t_{rmax}	12.98	[in]
				T_{rmax}	12.98	[%]
Tip airfoil thickness				t_{tmax}	3.16	[in]
				T_{tmax}	6.15	[%]



…TO BE CONTINUED